**Modelling Okun’s Law in the Euro Area –**

**Does non-Gaussianity Matter?**[[1]](#footnote-1)\*

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Abstract

In this paper, we analyse Okun’s law – a relation between the change in the unemployment rate and GDP growth – using euro area data ranging from 1995Q2 to 2019Q4. More specifically, we assess the relevance of non-Gaussianity when modelling the relation. This is done in a Bayesian VAR framework with stochastic volatility where we allow the different models’ error distributions to have heavier-than-Gaussian tails and skewness. Our results indicate that accounting for heavy tails – but not skewness – can provide improvements over a Gaussian specification.

*JEL Classification:* C11; C32; C52; E32

*Keywords*: Bayesian VAR; Heavy tails; GDP growth; Unemployment

## 1. Introduction

Okun’s law is a key macroeconomic relation which has become a popular tool for analysis and forecasting since its introduction almost sixty years ago (Okun, 1962). Typically relating the change in the unemployment rate to GDP growth, a substantial literature has analysed various aspects of it;[[3]](#footnote-3) see, for example, Knotek (2007), IMF (2010), Ball *et al*. (2017), An *et al*. (2019), Ball *et al*. (2019) and Karlsson and Österholm (2020). While conclusions from the empirical literature regarding the properties of Okun’s law differ somewhat depending on the country and period studied, Ball *et al*. (2017, p. 1439) nevertheless suggest that Okun’s law “… *is strong and stable by the standards of macroeconomics*”.

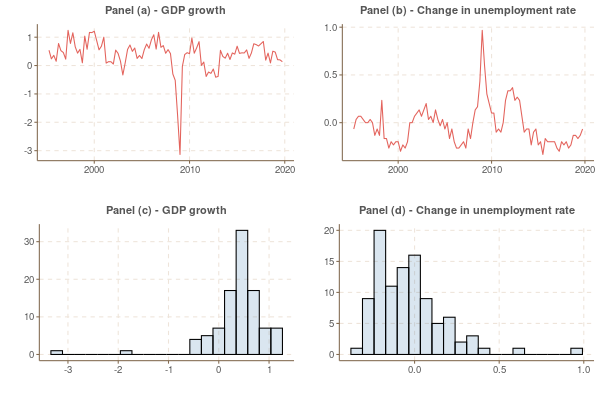
In this paper we contribute to the literature on Okun’s law by investigating the importance of non-Gaussianity when modelling the relation. More specifically, we address a fact that has received increasing attention in the macroeconomic literature, namely that many variables appear to be characterised by heavy tails (or “fat tails”) and skewness; see, for example, Acemoglu and Scott (1997), Fagiolo *et al*. (2008), Ascari *et al*. (2015), Bekaert and Popov (2019), Liu (2019) and Kiss and Österholm (2020). Heavy tails and skewness in the data can be caused by the disturbances of the model having these properties. Using data for the euro area, we assess this issue by estimating bivariate Bayesian VAR models with stochastic volatility under different assumptions regarding the error distributions and conducting formal model comparison based on the marginal likelihoods of the estimated models. We conclude that accounting for heavier-than-Gaussian tails finds support, whereas modelling skewness does not.

## 2. Data and empirical analysis

We use euro area data which range from 1995Q2 to 2019Q4.[[4]](#footnote-4) GDP growth () is given as the percentage change in seasonally adjusted real GDP from the previous quarter; the change in the seasonally adjusted harmonized unemployment rate () is given in percentage points. Data are shown in Figure 1 and some key descriptive statistics are given in Table 1.

As can be seen from Figure 1 and Table 1, the unconditional distribution of the variables seems to be slightly skewed, but more importantly, heavily leptokurtic. The Jarque-Bera test strongly rejects normality of both variables, which gives an initial indication that a departure from a Gaussian distribution might prove useful when modelling the Okun’s law relationship empirically.

Figure 1. Data.



Note: Panel (a) has percent and panel (b) percentage points on the vertical axis. Panel (c) has percent and panel (d) has percentage points on the horizontal axis; panels (c) and (d) both have frequency on the vertical axis.

Table 1. Descriptive statistics and Jarque-Bera test statistics.

|  |  |  |
| --- | --- | --- |
|  | GDP growth | Change in  unemployment rate |
| Mean | 0.39 | -0.03 |
| Standard deviation | 0.57 | 0.21 |
| Skewness | -2.93 | 1.55 |
| Kurtosis | 17.98 | 7.13 |
| Jarque-Bera | 1066.56 | 110.03 |
|  |  |  |

Note: The critical value at the five percent level of the Jarque-Bera test is 5.99.

Like Karlsson and Österholm’s (2020) analysis on US data, we rely on bivariate Bayesian VAR(1) models with stochastic volatility for our analysis of Okun’s law. Unlike Karlsson and Österholm though, we do not allow for time-variation in parameters and, importantly, we have flexible error term distributions that allow for heavy tails and skewness. Denoting we have

(1)

where is a vector of intercepts and includes the dynamic coefficients of the VAR. The error term follows either a multivariate skew-t distribution (m.skew-t)

, (2)

or an orthogonal skew-t distribution (o.skew-t)

, (3)

where the lower triangular matrix contains the structural parameters of the VAR model, is a diagonal matrix of independent mixing variables drawn from an inverse-gamma distribution with identical scale and shape parameters equal to , is the degree of freedom, is the vector of skewness parameters and . Equation (2) shows that the innovation has heavy tails and skewness and it correlates with the other innovations in the VAR while equation (3) considers the innovation as a linear combination of the skewed shocks. The matrix contains the stochastic volatilities of the variables, whose time series evolution is described as

(4)

for with and . Finally, , and are mutually independent. The distributions in (2) and (3) allow for both leptokurtic and skewed innovation distributions even after filtering out stochastic volatility; for details, see Karlsson *et al*. (2021). Setting yields the multivariate-t (m.t) and orthogonal-t (o.t) distributions, respectively. The Gaussian distribution is also nested in these specifications for ). We accordingly consider five BVAR models with stochastic volatility: the benchmark Gaussian, the multivariate-t and orthogonal-t with heavy-tailed innovations and the multivariate-skew-t and orthogonal-skew-t with both heavy tails and skewness.

Bayesian estimation requires specifying prior distributions for the parameters. We use a diffuse normal prior (with zero mean and variances 10) for the elements of the lower triangular matrix **A**. We impose a Minnesota prior for the regression coefficients ( and ) with overall shrinkage and cross-variable shrinkage (Koop and Korobilis, 2010). The priors for the rest of the parameters are given by, , and , where is a gamma distribution with parameters and (Kastner and Frühwirth-Schnatter, 2014).

Posterior means of the heavy-tail parameter and skewness are collected in Table 2, along with their credible intervals. The marginal likelihoods suggest that heavy tails should be taken into account. Both the orthogonal-t and the multivariate-t models have higher marginal likelihoods than the Gaussian model, where the orthogonal-t model is the overall preferred. The evidence is not overwhelming though; using the scale of two times the difference in log marginal likelihood and the terminology of Kass and Raftery (1995), it is “*not worth more than a bare mention*”. This is also reflected in the estimated degrees of freedom which ranges from 19 to 30 depending on the specifications. Regarding skewed error distributions, this finds no support in the data. Not only do the credible intervals for the skewness parameters include zero for both variables in both specifications, the marginal likelihoods also deteriorate even compared to the benchmark Gaussian model.

**Table 2. Non-normality parameters and marginal likelihoods from estimated models.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | **Gaussian** | **o.t** | **m.t** | **o.skew-t** | **m.skew-t** |
|  |  |  | 19.625 | 24.987 | 28.630 | 28.762 |
|  |  |  | (6.232 , 37.561) | (9.800 , 44.006) | (12.800 , 48.518) | (13.632 , 47.819) |
|  |  |  | 23.408 | 23.347 | 29.947 | 29.936 |
|  |  |  | (9.587 , 41.137) | (8.838 , 42.375) | (12.396 , 51.630) | (13.419 , 50.290) |
|  |  |  |  |  | 0.146 | 0.383 |
|  |  |  |  |  | (-0.394 , 0.660) | (-0.137 , 0.792) |
|  |  |  |  |  | 0.016 | 0.011 |
|  |  |  |  |  | (-0.085 , 0.120) | (-0.082 , 0.105) |
|  | *Log marginal likelihood* | 24.856 | 25.572 | 25.229 | 21.770 | 21.595 |
|  |  |  |  |  |  |  |

Note: Parameter posterior means are presented along with their 80 percent credible interval. gives the degrees of freedom for the errors of variable i and the skewness parameter.

Using the best performing model – that is, the orthogonal-t model – we finally illustrate a key aspect, namely how the change in the unemployment rate responds to an unexpected increase in GDP growth; Figure 2 presents this impulse response.[[5]](#footnote-5) As can be seen, the impulse response is negative contemporaneously and remains negative over the entire ten-quarter horizon, in line with what is expected. It can be noted that the magnitude of the impulse response changes considerably over the sample period. Since the dynamic parameters of the model are constant, the only source of this variation is the stochastic volatility of GDP growth.

Figure 2. Impulse response function – response of change in unemployment rate to a shock in GDP growth.

Chart, surface chart

Description automatically generated

Note: The impulse response function is based on the model with orthogonal-t distributed errors. Percentage points on the vertical axis. Horizon is given in quarters on the horizontal axes.

To see this variation in volatility, we present the posterior mean of the estimated standard deviation of the shocks to both variables in Figure 3. The volatility of GDP growth is relatively low during most of the sample period. To some extent this is not surprising as part of our sample reflects “the Great Moderation”, a period where macroeconomic aggregates exhibited historically low volatility. The period of the 2008-2009 financial crisis constitutes an exception, where the volatility of the GDP growth peaks. This means that large shocks were hitting the economy during this period, which in turn leads to the large movements in the magnitude of the impulse responses in Figure 2.

Figure 3. Estimated standard deviation of shock to GDP growth and change in unemployment rate.

Chart, line chart

Description automatically generated

Note: Percentage points on vertical axis.

## 3. Conclusion

In this paper, we have analysed the relevance of taking non-Gaussianity into account when empirically modelling Okun’s law in the euro area. Our results based on Bayesian VAR models with stochastic volatility suggest that heavier-than-Gaussian tails find support. While the evidence is not excessively strong, the model based on the orthogonal-t distribution is ranked first according to marginal likelihoods. Taking skewness into account is, however, not beneficial in this context.

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3. Another way to specify the relation is to connect the unemployment rate (or unemployment gap) to the output gap. [↑](#footnote-ref-3)
4. We do not use data from 2020 since the corona pandemic induced movements in the variables – particularly GDP growth – some of which were so large that we deem them to be outliers (and accordingly not suitably accounted for by heavy tails). [↑](#footnote-ref-4)
5. The other impulse responses of the model are omitted due to space constraints. [↑](#footnote-ref-5)